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OPTIMIZATION OF CROSS-SECTIONAL AREA OF THE MANIPULATOR ARM

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ABSTRACT: It remains the designer's main task to incorporate in the designed object such parameters which will be in harmony with the requirements of its practical application and which will take into account its mass, shape, geometry, and possibly of its some dynamic properties. The main aim usually is material saving and finding the best solutions not only in terms of the machine operation in the production process, but also in terms of an adequate material utilization and in terms of a suitable shape of the designed structure.

KEY WORDS: Optimization, analysis, finite elements method

1. INTRODUCTION

Stricter requirements placed at present on saving of materials, long service life, reliable and safe operation of mechanical equipment call for new approaches in putting various technical ideas into practice. It is possible, with the use of suitable computing systems, to produce a fast and efficient study of the parameters which in a greater or a lesser extent influence the static or dynamic characteristics of a machine.

1.1 General formulation of the optimization problem

In the first phase is necessary to create correct physical machine model (static or dynamic). Second phase requires investigation model description by the proper mathematic means, it is creation of the mathematic model (equations system, couplings conditions, etc.) The third and often most difficult phase of optimal construction designing is mathematic formulation of the optimization problem. This we can formulate like extreme research (minimum) of the objective function:

$$z = F(\{X\}), \tag{1}$$

by additional (limiting) conditions which are given by irregularities which determinate admissible area:

$$G_j(\{X\}) \leq \{0\} \tag{2}$$

where $\{X\} = [X_1, X_2 \ldots X_n]^T$ is vector of designing (design) variables. Equations (1) and (2) contains general non-linear functions of designing parameters $\{X\}$. They belong to problems of mathematic programming, often non-linear. For research of global minimum we use Kuhn-Tucker obligatory conditions, which we can specify:

$$\nabla F(\{X\}) + \sum_{j=1}^{m} \lambda_j \cdot \nabla G_j(\{X\}) + \sum_{k=1}^{l} \Lambda_{m+k} \cdot \nabla H_k(\{X\}) = \{0\} \tag{3}$$

Mathematic programming belongs to direct methods of optimization problems solution. In consequence of general non-linear character of objective function and limitation functions will equations (1), (2), (3) be non-linear. This state can not be in majority cases solved analytical. Then it is proper to apply iterative methods, which allow compute vectors chain $\{X\}_r$ (r = 1, 2.....), which converge to optimum $\{X\}_{opt}$. Direct equations application (3) in optimization process impacts to problems with limitations in non-equations shape and therefore are used other methods of non-linear programming. Biggest application in optimization problems gain methods of "zero" place value (comparator) and methods of "first" place value (gradient). More effective are methods gradient. Their general disadvantage is big operations number by analyse of sensitivity. Basically it deals about various modified iterative methods of Newton type. General idea of these iterative approaches we can express by equation:

$$\{X\}_{k+1} = \{X\}_k - \alpha \cdot \nabla F(\{X\}_k) \tag{4}$$

Operator ∇ expresses just like in equation (3) gradient of objective function in k-th iterative step for direction of progress optimum finding.

It has been shown in practice that direct optimizing methods involve rather demanding computations and to a major portion of technicians they may appear as too complicated. However, there exist indirect methods that do not operate directly with the target function and that evaluate only the limitations (usually stresses). One of such methods is the Fully stress design, and we will utilize some of its basic principles and procedures in our reasoning below.

The method is based on the idea of designing the structural variables in such a manner that the stresses in the entire structure will be at the permissible limit.

1.2 The method of "Fully Stress Design"

This method seeks to find an optimal solution when utilizing the overall loading capacity of the material at the given load. The limiting conditions for the stresses may be written as follows:

$$\sigma_i - \sigma_{iperm} \leq 0, \tag{5}$$

where σ_i is the stress at the given point and σ_{iperm} is the maximal permitted stressing. As an example, an optimal design of beam structures will be used with respect to their sectional characteristics.

1.3 Optimization of sectional parameters by beam elements

We consider duo node beam finite element of constant cross-section. In this element are slide forces, axial forces, torque constant and elastic moments are changing linear between both ending nodal points. Maximal stress will be at one or other end of beam, therefore points of computed stresses will be there. Except this we must define also points position, in which we compute stresses in cross-sectional surface. Final stresses we compute according to equations:

- axial compound of normal stress:

$$\sigma_x = \frac{F_x}{A} - \frac{M_y}{J_y}.C_z + \frac{M_z}{J_z}.C_y \tag{6}$$

- slide stress from slide forces:

$$\tau_1 = \frac{F_y.S_y}{A}$$
$$\tau_2 = \frac{F_z.S_z}{A} \tag{7}$$

- slide stress from torque:

$$\tau_3 = \left(M_x + F_z \cdot e_y - F_y \cdot e_z\right) \cdot \frac{r_T}{J_k}, \tag{8}$$

where F_x, F_y, F_z are compounds of internal force in given cross-section, M_x, M_y, M_z are internal moments, A is cross-sectional surface, J_y and J_z are axial quadratic moments of persistence, J_k is cross-sectional torque moment, S_y and S_z are static moments of cross-sectional part dedicated by point area, in which we compute stress, c_y and c_z are point distances in section plane, in which we compute stresses from cross-sectional centre of gravity, e_y and e_z are coordinates of eccentricity (distance between centre of gravity and slide centre), r_T is effective radius of torsion.
Equivalent stress is computed according to HMH hypothesis (von Mises):

$$\sigma_{ekv} = \sqrt{\sigma_x^2 + 3 \cdot \left((\tau_1 + \tau_2)^2 + \tau_3^2\right)} \tag{9}$$

Computational points for beam cross-section must be defined there, where we can expect extreme values.

To add we define stiffness matrix of three-dimensional beam finite element and calculation of internal force parameters, from which will be assigned tenseness in endpoint nodes according to equations (6), (7), (8).

If axis y, z will be central, then $S_y = 0$, $S_z = 0$, $D_{yz} = 0$ and we obtain known, in literature very often published, stiffness matrix of beam element.
There are many approaches in optimization of beam cross-section. One approach can be optimization of total height, width or both cross-sectional variables. In this way works optimization modulus in Cosmos program.

1.4 Optimization of cross-sectional area of the manipulator arm

Optimization of the manipulator arm was done in program COSMOS 2.85 and by optimal designing we came out from model (Fig. 2), where was optimized whole width of beam arm.

Fig. 1: Manipulator arm

Arm s cross-sectional area of individual beam elements was changing with element wall width and element wall height was constant. Stress limitation value was specified at σ_{idov} = 150 [MPa]. The arm was loaded by own weight and maximal (standard) weight value of risen load of 20 000 N at one and second side of arm. Arm weight before optimization was 48.9 kg.
Tenseness distribution in manipulator arm before optimization is at fig. 3.

Fig. 2: Tenseness distribution before optimization in [MPa]

Tenseness distribution in manipulator arm after optimization is at fig. 4.

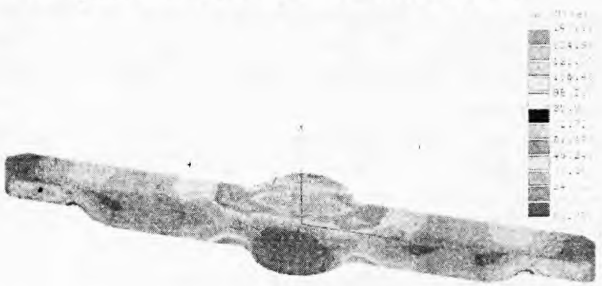

Fig. 3: Tenseness distribution after optimization in [MPa]

Optimization result in individual iterative steps is at fig. 5.

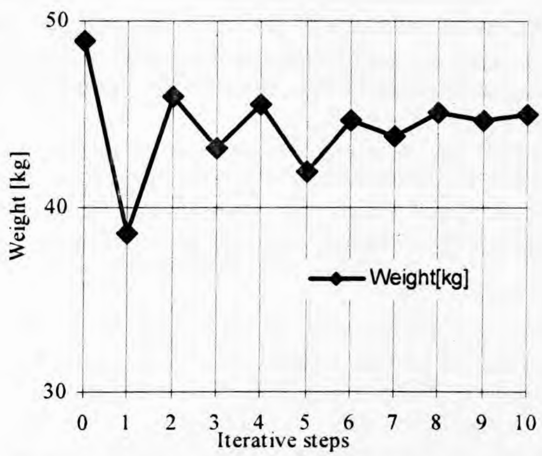

Fig. 4: Convergence graph of objective function

As we can see from fig. 5, minimal arm weight was reached in tenth iterative step. We reduced arm weight about 3,975 kg.

3. CONCLUSION

The Finite Element Method enables to solve many complex practical tasks that in the past were almost insolvable. It cannot be generally maintained that some particular method is the best and always leads to finding the optimal solution. Each problem is peculiar in its own way due to its physical or mathematical model or due to the procedure used in the finding of the best solution. Therefore, in finding the effective procedure in the optimization of mechanically stressed structures, the following should be taken into account:

- peculiarities of the problem to be solved (static or dynamic task, linear or non-linear behaviour of the structure),
- the way of stressing and the number of loading modes,
- the number of design variables and the character thereof,
- expected location of the optimum with regard to the limitations.

It is necessary to note, however, that designers cannot consider purely static optimizing analysis as the final design before dynamic analysis is carried out. The important point here is that the designer should be aware of the influence of the changes of rigidity and mass of the structure upon its dynamic characteristics. For example in transport machines a reduction in flexural rigidity and the subsequent reduction in mass may result in a sharp increase in the amplitudes of vibrations already at lower frequencies due to which resonance will occur and operational properties of the equipment will decrease, or places with stress peaks will be formed which lead to damaging the structure (formation and growth of cracks, etc.).

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